9. (12 marks) A and B are 3×3 matrices and $\det A = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 4$, and $\det B = 3$. Find

       a) $\det \left(A^T (2B)^3 \det(B) \right)$ b) $\det \left(5BA^{-1} - 2B \, adj(A) \right)$

       c) $\begin{vmatrix} 2a & 3g & 5g & 4a & d \\ 2b & 3h & 5h & 4b & e \\ 2c & 3i & 5i & 4c & f \end{vmatrix}$

10. (4 marks) Evaluate the determinant $\begin{vmatrix} 2 & 1 & 5 & 1 \\ 8 & 0 & 1 & 3 \\ 1 & 1 & 6 & 2 \\ 3 & 1 & 5 & 3 \end{vmatrix}$ by row reduction. **You must perform at least one row operation.**

11. (9 marks) Let $\vec{u} = \langle 2, 1, 3 \rangle$, $\vec{v} = \langle 1, 2, 4 \rangle$, $\vec{w} = \langle 3, 1, \rangle$.

a) Find the orthogonal projection of the vector $\vec{w}$ on the vector $\vec{u} \times \vec{v}$, that is $Proj_{\vec{u} \times \vec{v}} \vec{w}$.

b) Find a unit vector perpendicular to $\vec{u} \times \vec{v}$ and $\vec{w}$.

c) Find the area of a triangle determined by $\vec{u} \times \vec{v}$ and $\vec{w}$.

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Answers

1. a) $x_1 = 2 - 3t - s,\ x_2 = 1 + t + 5s,\ x_3 = t,\ x_4 = s.$ b) $x_1 = 1,\ x_2 = 4,\ x_3 = 0,\ x_4 = 1.$

2. a) $A^{-1} = \begin{pmatrix} 1 & \frac{1}{3} & \frac{5}{3} \\ 1 & 0 & 1 \\ 1 & \frac{2}{3} & \frac{4}{3} \end{pmatrix}$; b) $x = 1,\ y = 2,\ z = 1.$

3. $y = 2$

4. $4A^{-2}$

5. a) -47;   b) $X = \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}$

6. $E_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $E_2 = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$. *Other possible answers.*

7. 1) $k \ne 3$; 2) $k = 3$; 3) $k \ne 3$

8. False.

9. a) $\frac{1}{128}$ b) $\frac{81}{4}$; c) 40

10. -16

11. a) $\frac{51}{ },\ \frac{51}{ },\ \frac{17}{ }$; b) $\frac{7}{\sqrt{94}}, \frac{6}{\sqrt{94}}, \frac{3}{\sqrt{94}}$; c) $\sqrt{94}$.

12. a) Yes, the line is parallel to the plane;   b)   ;   c) $2\sqrt{14}$.

13. $x = 11 + 13t,\ y = -3 - 2t,\ z = t.$

14. 0

15. b) $29x - 3y - 13z - 118 = 0$

16. $P = 43,\ x_1 = 3,\ x_2 = 0,\ x_3 = 4,\ x_4 = 0,.$

17. $C = 60,\ x_1 = 0,\ x_2 = 20,\ x_3 = 0.$