(6) + (7) + (8)

$$y'' + y' + y = e^{2x}\{(-5+4+1)\sin 3x + (12+6)\cos 3x\} = 18\cos 3x$$

Assignment for functions satisfying D.E.

(1) $y = 2x^3 - x^2;\ xy' - 3y = x^2.$
(2) $y = \frac{1}{2}\cos x;\ y'' + 9y = 4\cos x.$
(3) $y = \sin x + \cos x - e^{-x};\ y' + y = 2\cos x.$
(4) $y = 3e^{-2x} + 4e^{-x};\ y'' + 3y' + 2y = 0.$
(5) $y = t\sin 2t;\ y'' + 4y = 4\cos 2t.$
(6) $y = 5xe^{-4x};\ y'' + 8y' + 16y = 0.$
(7) $y = e^{-3x}\sin 2x;\ y'' - 5y = -12e^{-3x}\cos 2x.$

Applications of differential equations

(1) When a hockey player strikes a puck with a certain force at $t = 0$ the puck moves along the ice with velocity at any time thereafter given by the expression $v(t) = 27 - 9\sqrt{t}$ *m/sec.* How far would the puck travel on a long sheet of ice before coming to rest? (Asumme $s(t) = 0$ at $t = 0$.)
Solution: $\frac{ds}{dt} = v(t)$. Therefore

$$s = \int ds = \int v(t)dt = \int (27 - 9\sqrt{t})dt = 27t - \frac{9t^{3/2}}{3/2} + C$$
$$= 27t - 6t^{3/2} + C.$$

At $t = 0$, $s(t) = 0 = 27(0) - 6(0) + C$. Therefore $C = 0$. When the puck stops, $v(t) = 0 = 27 - 9\sqrt{t}$. Hence $9\sqrt{t} = 27$ so $\sqrt{t} = 3$. Therefore $t = 9$ *sec.* Substituting in the equation for s, $s(9) = 27(9) - 6(9)^{3/2}$

Solution: (a) Separating the variables we have

$$\frac{dv}{-v^2} = 0.04\ dt \quad \text{or} \quad \int -v^{-2}dv = \int 0.04dt.$$

$\frac{-v^{-1}}{-1} = 0.04t + C$ or simplifying $\frac{1}{v} = 0.04t + C$. Now applying the initial conditions, when $t = 0$, $v = 10\ m/sec.$ so $1/10 = C.$ Therefore

$\frac{1}{v} = 0.04t + \frac{1}{10} = \frac{0.4t + 1}{10}$. Inverting the equation $v = \frac{10}{1 + 0.4t}$.

(b) When $t = 10\ sec.$,

$$v = \frac{10}{1 + (0.4)(10)} = \quad 10$$

(4) a train is travelling at 64 $km/hr.$ when the caboose suddenly becomes detached. If the friction of the rails provides a deceleration of $a = \frac{dv}{dt} = -2v^{3/2}\ km/hr.^2$, how fast will the caboose be going after $\frac{1}{4}$ of an hour? (Hint: Solve the D.E. $-\frac{dv}{v^{3/2}} = 2dt$)

(5) A population of whales grows at the rate $\frac{dP}{dt} = \frac{2}{3}P^{1/4}$. If $P = 16$ when $t = 0$ years, how many whales will there be after 38 years?

(6) The volume of water in a tank is given by $V = 8h^{3/2}$. Therefore

$$h^{3/2} = \frac{V}{8} \quad \text{or} \quad h = \left(\frac{V}{8}\right)^{2/3} = \frac{V^{2/3}}{4}.$$

If the water drains from the tank according to Toricelli's law, then

$$\frac{dV}{dt} = -2\sqrt{h} = -2\sqrt{\frac{V^{2/3}}{4}} = -2\frac{V^{1/3}}{2} = -V^{1/3} \qquad \frac{dV}{V^{1/3}} = -dt.$$

(a) If the tank is initially filled with 125 m^3