Name: SOLUTIONS

Student ID Number: _____

Teacher's name: _____

Course name: _____

Section number: _____

Final Examination Rules

Before you begin your exam:

→ The use/possession of cell phones and other elect
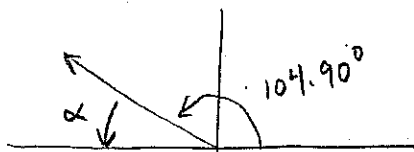
Dawson College

Department of Mathematics

Final Examination
Winter 2010

Introduction to Applied Mathematics (201 -912-DW)

Date: December 2010   Time: 3 h

Examiners: G. Honnouvo, J. Requeima

$$\cos^{-1}(-0.2572) = 104.90°$$

$$\alpha = 180° - 104.90° = 75.10°$$

104.90°

α
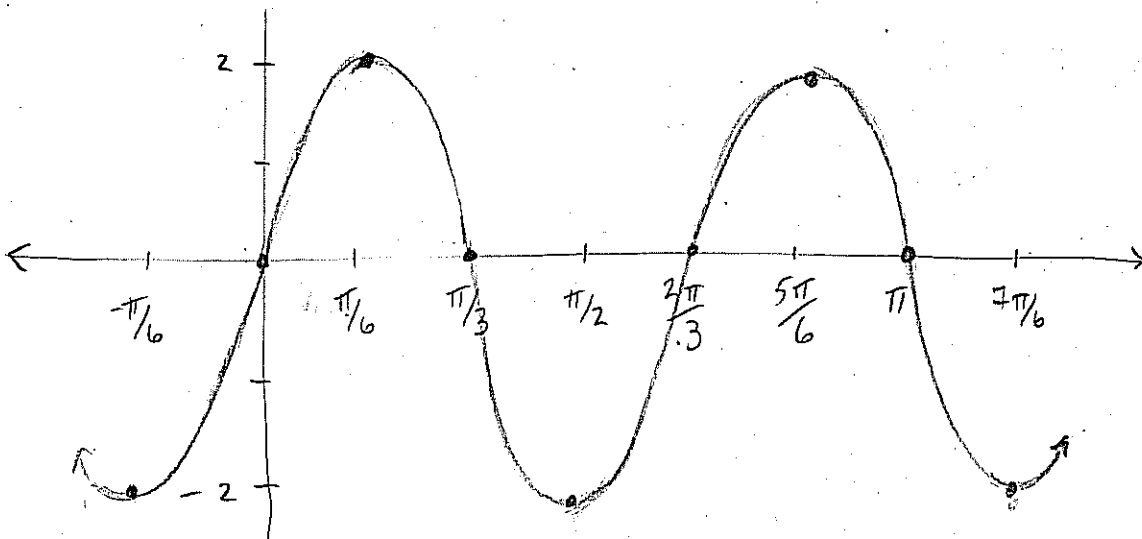
Question 2. *(8 marks)* Graph $y = -2\sin(3x - \pi)$.

AMPLITUDE: $|a| = 2$

PERIOD $= \dfrac{2\pi}{3}$

PHASE SHIFT $= -\dfrac{c}{b} = -\dfrac{(-\pi)}{3} = \dfrac{\pi}{3}$

x	$\pi/3$	$\pi/2$	$\dfrac{2\pi}{3}$	$\dfrac{5\pi}{6}$	π
y	0	-2	0	2	0

Question 3. *(8 marks)* Find the components, magnitude and standard position angle of the sum R of $A = 237$, $\theta_A = 137.5°$ and $B = 121$, $\theta_B = 196.4°$

$A_x = A\cos\theta_A = 237\cos 137.5° = -174.735$
$A_y = A\sin\theta_A = 237\sin 137.5° = 160.115$

$B_x = B\cos\theta_B = 121\cos 196.4° = -116.077$
$B_y = B\sin\theta_B = 121\sin 196.4° = -34.163$

$\therefore R_x = A_x + B_x = -174.735 - 116.077 = -291$

$R_y = A_y + B_y = 160.115 - 34.163 = 126.$

$\therefore R = \sqrt{(-291)^2 + (126)^2}$

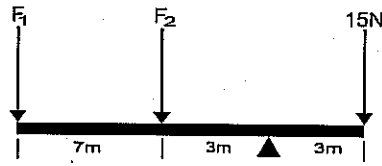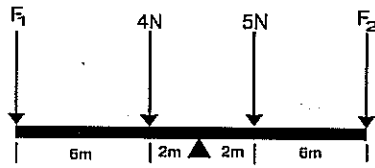$\tan\theta_R = \dfrac{126}{-291} \Rightarrow \tan^{-1}\left(\dfrac{126}{-291}\right) \approx 23.4° = \alpha$

θ_R IS IN QUAD II

$\theta = 180° - 23.4°$

$= 156.6°$

Question 4. *(8 marks)* Use Cramer's rule to solve for F_1 and F_2 in the systems below that are in equilibrium. (Round to 3 significant figures)

$$8F_1 + 2(4) = 2(5) + 8F_2$$

$$8F_1 - 8F_2 = 10 - 8$$
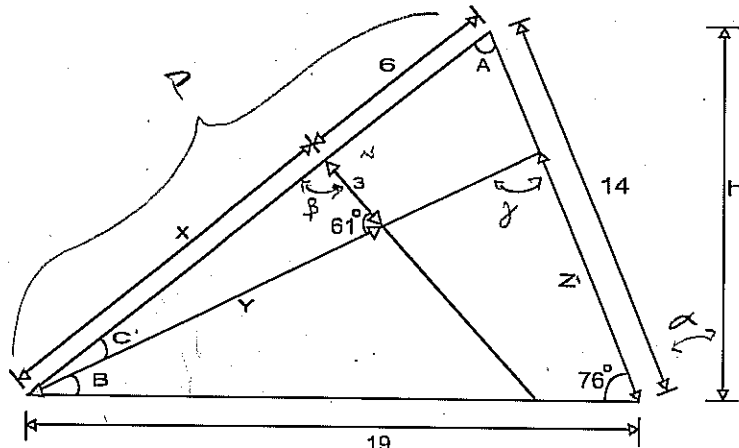
$$8F_1 - 8F_2 = 2$$

$$10F_1 + 3F_2 = 3(15)$$

$$10F_1 + 3F_2 = 45$$

$$F_1 = \frac{\begin{vmatrix} 2 & -8 \\ 45 & 3 \end{vmatrix}}{\begin{vmatrix} 8 & -8 \\ 10 & 3 \end{vmatrix}} = \frac{6 + 360}{8(3) - (-8)(10)} = \frac{366}{104} = 3.52N$$

$$F_2 = \frac{\begin{vmatrix} 8 & 2 \\ 10 & 45 \end{vmatrix}}{\begin{vmatrix} 8 & -8 \\ 10 & 3 \end{vmatrix}} = \frac{8(45) - 2(10)}{104} = \frac{340}{104} = 3.27N$$

Question 5. *(10 marks)* Determine the height h and the panel lengths X, Y and Z of the roof truss. Also find the angles A, B, and C. (Round to 3 decimal places)

$$\alpha = 90°-76° = 14° \quad\Rightarrow\quad \cos 14° = \frac{h}{14} \quad\Rightarrow\quad h = 14\cos 14° = \underline{13.584}$$

$$P^2 = 19^2 + 14^2 - 2(19)(14)\ldots 710$$

$$= 428.297515$$
$$\therefore P = 20.695$$

$$= 20.695 - 6$$
$$= \underline{14.695}$$

$$\frac{14.695}{\operatorname{sm} 61°} = \frac{3}{\operatorname{sm} C} \quad\Rightarrow\quad \operatorname{sm} C = \frac{3\operatorname{sm} 61°}{14.695} \quad\Rightarrow\quad \underline{C = 10.286°}$$

$$\beta = 180° - 10.286° - 61° = 108.714°$$

$$\frac{Y}{\operatorname{sm} 108.714°} = \frac{14.695}{\operatorname{sm} 61°} \quad\Rightarrow\quad \underline{Y = 15.913}$$

$$\frac{19}{\operatorname{sm} A} = \frac{20.695}{\operatorname{sm} 76°}$$

$$\Rightarrow \quad \operatorname{sm} A = \frac{19 \operatorname{sm} 76°}{20.695}$$

$$\Rightarrow \quad A = 62.977°$$

$$\therefore B = 180° - (12.02)° - 76°$$

Question 6. *(10 marks)*
(a) Evaluate $\sqrt{224}(3167)^{167} = X$

$\log x = \log \sqrt{224}\,(3167)^{167} = \frac{1}{2}\log 224 + 167\log 3167$

$$= 585.7834$$

$$\therefore X = 10^{585.7834} = 10^{585}\,10^{0.7834} = 6.07 \times 10^{585}$$

(b) Solve for x in $\log(x-1)+\log(3x+2)=2$

$$\log(x-1)(3x+2) = \log 10^2$$

$$(x-1)(3x+2) = 100$$

$$3y^2 - 3x + 2x - 2 = 100$$

$$3x^2 - x - 102 = 0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-102)}}{2(3)} = \frac{1 \pm \sqrt{1225}}{6} = \frac{1 \pm 35}{6}$$

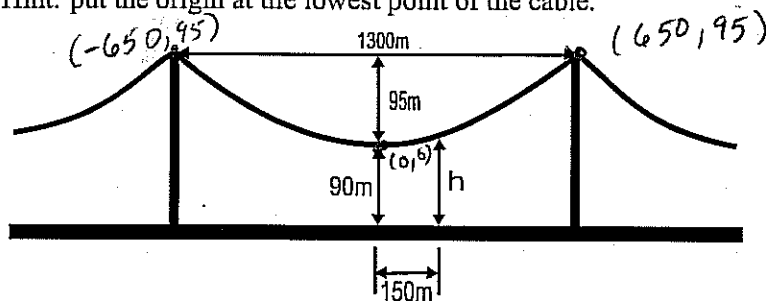$$= -5\cancel{\frac{}{}7} \;,\; 6$$

Question 7. *(5 marks)* In curing concrete the strength f_c after t days of curing is given by the equation

$$f = f_c\left(1 - e^{-kt}\right)$$

where $f_c = 48$Mpa is the ultimate strength. If $k = 0.1045$ how long does it take for concrete to achieve 75% of its ultimate strength.

the height h of the cable 150m from the lowest point of the cable? (Round to 4 significant figures).
Hint: put the origin at the lowest point of the cable.

$(-650, 95)$

1300m

$(650, 95)$

95m

$(0, 6)$

90m

h

150m

$y = ax^2 + bx + c$

$0 = a(0)^2 + b(0) + c \implies c = 0$

$95 = a(650)^2 + b(650) \implies 95 = 422500a + 650b$

Question 9. *(8 marks)* Find the point(s) of intersection of the line that passes through the point $(1,3)$ with slope $m = 3/4$ and the circle $2x^2 + 2(y-4)^2 = 50$. Is this line a tangent line, secant line, or neither?

$y = mx + b \qquad \Rightarrow \qquad \frac{12 - 3}{4} = b \qquad \qquad \qquad$

$x^2 + (y-4)^2 = 25$

$x^2 + \left(\frac{3}{4}x + \frac{9}{4} - 4\right)^2 = 25$

$x^2 + \left(\frac{3}{4}x - \frac{7}{4}\right)^2 = 25$

$x^2 + \frac{9}{16}x^2 - \frac{42}{16}x + \frac{49}{16} = 25$

$16x^2 + 9x^2 - 42x + 49 = 400$

$25x^2 - 42x - 351 = 0$

Question 10. *(8 marks)* In testing an air-conditioning system that was recently installed in a building, the temperature in a building was measured during the afternoon hours with the results

below. Find the least squares-line and graph this line over a plot of the data points. What would you expect the temperature to be in the building at 6 hours?

| time in hours (x) | 0.0 | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 |